8.6 Applications of Derivative

Functions: f, g, y
Position of an object: s
Velocity: v
Acceleration: w
Independent variable: x
Time: t
Natural number: n

825. Velocity and Acceleration
$s = f(t)$ is the position of an object relative to a fixed coordinate system at a time t,
$v = s' = f'(t)$ is the instantaneous velocity of the object,
$w = v' = s'' = f''(t)$ is the instantaneous acceleration of the object.

826. Tangent Line
$y - y_0 = f'(x_0)(x - x_0)$

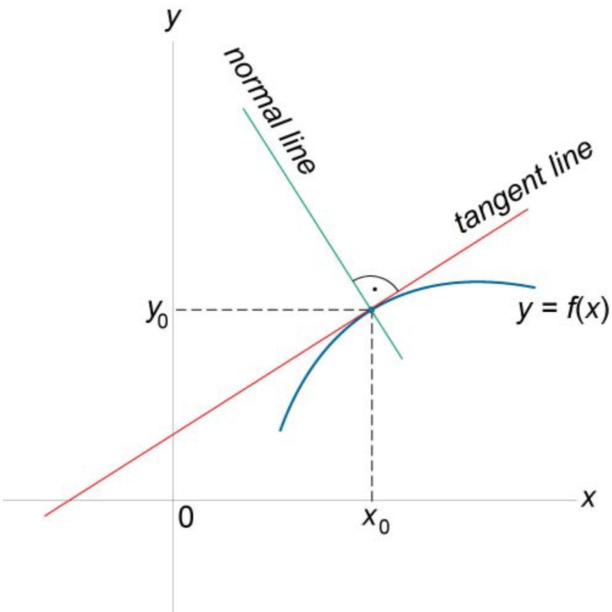

Figure 176.

827. Normal Line

$$y - y_0 = -\frac{1}{f'(x_0)}(x - x_0) \text{ (Fig 176)}$$

828. Increasing and Decreasing Functions.

If $f'(x_0) > 0$, then f(x) is increasing at x_0. (Fig 177, $x < x_1$, $x_2 < x$),

If $f'(x_0) < 0$, then f(x) is decreasing at x_0. (Fig 177, $x_1 < x < x_2$),

If $f'(x_0)$ does not exist or is zero, then the test fails.

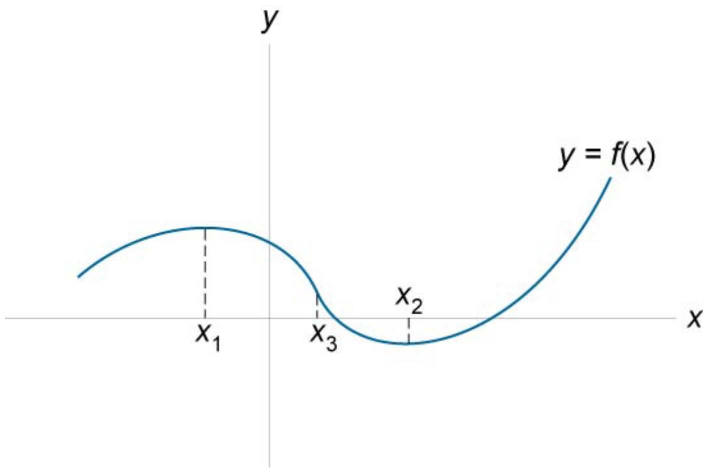

Figure 177.

829. Local extrema

A function f(x) has a local maximum at x_1 if and only if there exists some interval containing x_1 such that $f(x_1) \ge f(x)$ for all x in the interval (Fig.177).

A function f(x) has a local minimum at x_2 if and only if there exists some interval containing x_2 such that $f(x_2) \le f(x)$ for all x in the interval (Fig.177).

830. Critical Points

A critical point on f(x) occurs at x_0 if and only if either $f'(x_0)$ is zero or the derivative doesn't exist.

831. First Derivative Test for Local Extrema.

If f(x) is increasing $(f'(x) > 0)$ for all x in some interval $(a, x_1]$ and f(x) is decreasing $(f'(x) < 0)$ for all x in some interval $[x_1, b)$, then f(x) has a local maximum at x_1 (Fig.177).

832. If f(x) is decreasing ($f'(x)<0$) for all x in some interval $(a, x_2]$ and f(x) is increasing ($f'(x)>0$) for all x in some interval $[x_2, b)$, then f(x) has a local minimum at x_2. (Fig.177).

833. Second Derivative Test for Local Extrema.
If $f'(x_1)=0$ and $f''(x_1)<0$, then f(x) has a local maximum at x_1.
If $f'(x_2)=0$ and $f''(x_2)>0$, then f(x) has a local minimum at x_2. (Fig.177)

834. Concavity.
f(x) is concave upward at x_0 if and only if $f'(x)$ is increasing at x_0 (Fig.177, $x_3 < x$).
f(x) is concave downward at x_0 if and only if $f'(x)$ is decreasing at x_0. (Fig.177, $x < x_3$).

835. Second Derivative Test for Concavity.
If $f''(x_0)>0$, then f(x) is concave upward at x_0.
If $f''(x_0)<0$, then f(x) is concave downward at x_0.
If $f''(x)$ does not exist or is zero, then the test fails.

836. Inflection Points
If $f'(x_3)$ exists and $f''(x)$ changes sign at $x = x_3$, then the point $(x_3, f(x_3))$ is an inflection point of the graph of $f(x)$. If $f''(x_3)$ exists at the inflection point, then $f''(x_3)=0$ (Fig.177).

837. L'Hopital's Rule

$$\lim_{x\to c}\frac{f(x)}{g(x)}=\lim_{x\to c}\frac{f'(x)}{g'(x)} \text{ if } \lim_{x\to c} f(x)=\lim_{x\to c} g(x)=\begin{cases}0\\ \infty\end{cases}.$$